Fixed Point Analysis of Single Cell IEEE 802.11e WLANs: Uniqueness, Multistability and Throughput Differentiation

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Outline of Talk

1. Introduction
2. Fixed Point Based Saturation Analysis
3. Homogeneous Nodes: Uniqueness, Multistability, Unfairness
4. Nonhomogeneous Nodes

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IEEE 802.11 DCF, a form of CSMA/CA?

- We will focus on the Distributed Coordination Function (DCF) algorithm of IEEE 802.11: a form of CSMA/CA
- Why study such a MAC.... it is an old and well established technique?
- Many current and emerging systems utilise this MAC
- In addition to the IEEE 802.11 series of networks
  - Sensor networks: SMAC, and IEEE 802.15.4
  - Availability of inexpensive chip-sets is driving new applications
    - e.g., rural access networks based on long distance IEEE 802.11 links
- A variety of new questions are being asked
  - Service differentiation, delay analysis, admission control, power control
  - Enterprise WLAN planning; automatic channel allocation
  - Enterprise WLAN performance monitoring and management
    - How should "packet sniffer" measurements be used?
  - Multihop IEEE 802.11 networks

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Traffic Models Used in Our Analyses

- Nodes saturated: i.e., “infinite” backlog of packets
  - Saturation analysis is an important first step
  - Provides a measure of network capacity
    - Sometimes can be proven to be a sufficient stability condition
  - Directly useful in some modeling situations
  - By-products of saturation analysis have been found to work extremely well in unsaturated models (e.g., modeling with VoIP and TCP)
- Our work has appeared in: IEEE Infocom 2005, IEEE Sigmetrics 2005 IEEE/ACM Tr. on Networking 2007, and IEEE/ACM Tr. on Networking 2008
  - The present talk will summarise some of this work
- The following have been studied, but will not be reported in this talk
  - TCP controlled file transfers
  - Packet voice telephony (CBR packet streams)
  - Streaming video
  - Combinations of these traffic types

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Homogeneous vs. Nonhomogeneous

- Homogeneous: All nodes have the same access parameters
  - This is the situation in IEEE 802.11b
- Nonhomogenous: Nodes can have difference access parameters
  - This would be done to provide service differentiation
  - The IEEE 802.11e standard allows this
- We will begin with the analysis of the backoff process in the homogeneous case

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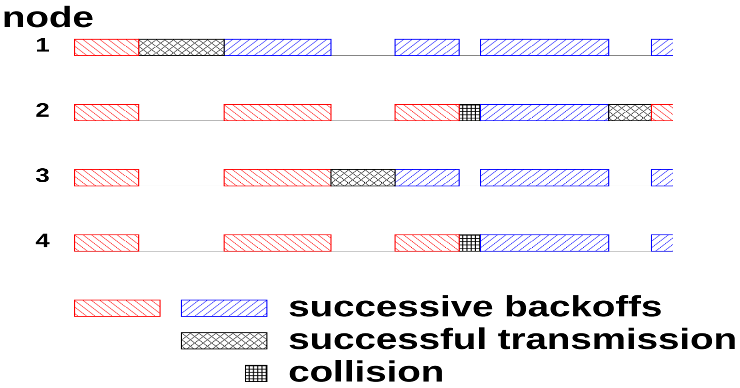
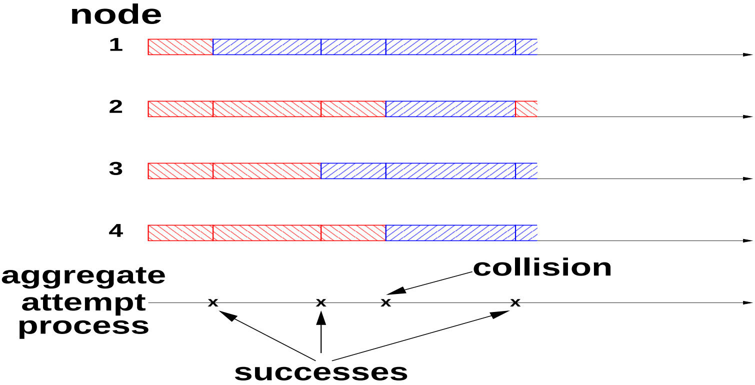
Saturated Nodes: Backoffs, Transmissions, Collisions



- Channel alternates between activity and back-off periods
- Backoff time is slotted and freezes during channel activity
- Activity period depends on which nodes cause the activity

Focusing Only on the Backoff Durations



- This evolution depends only on the back-off parameters, and the collision and success events
- The nodes' backoff evolutions are coupled due to collisions
- Can reconstruct original process from this process

A Detailed Markov Chain

- There are n nodes, with saturated queues
- Time evolves over system slotes, $t \in \{0, 1, 2, \cdots\}$
- Let $Z_i(t)$ be the backoff stage of node i in slot t
  - $Z_i(t) \in \{0, 1, \cdots, K\}$
- Let $Y_i(t)$ be the residual backoff of node i in slot t
- Then $\{((Z_1(t), Y_1(t)), (Z_2(t), Y_2(t)), \cdots, (Z_n(t), Y_n(t))), t \geq 0\}$ is a Markov chain
- Too complex to analyse, even if we take the backoff durations to be geometrically distributed

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Attempt and Collision Rates

- Let β be a node's attempt probability (in a backoff slot)
  - i.e., β is the attempt rate of a node, per slot, averaged over all back-off slots
  - Let $A_i(t)$ = number of attempts by Node i in $[0, t]$, then

$$\beta = \lim_{t \to \infty} \frac{A_i(t)}{t}$$

- Let γ be the collision probability seen by a node
  - i.e., γ is the fraction of a node's attempts that collide with other attempts
  - Let $C_i(t)$ = number of collisions seen by Node i in $[0, t]$, then

$$\gamma = \lim_{t \to \infty} \frac{C_i(t)}{A_i(t)}$$

- Thus, these are time average rates
- Two equations can be written relating these quantities
  - Bianchi, IEEE JSAC'00; our generalisation, IEEE Infocom'05, IEEE/ACM TON 2007

A Fixed Point Equation

- Isolate a single node and view the remaining $n-1$ as an "environment"
- "Decoupling" approximation
  - From the point of view of a node, the other nodes are viewed as attempting independently in each slot with probability β
- The node "response" equation
  - $\beta = G(\gamma)$: models how a node adjusts to collisions
- The coupling equation
  - $\gamma = \Gamma(\beta) := 1 - (1-\beta)^{n-1}$
  - Consequence of the "decoupling" approximation
- This yields the following, natural, fixed point equation

$$\gamma = \Gamma(G(\gamma))$$

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The Backoff Parameters for Each Node

- Homogeneous case
- The following are the back-off parameters for each packet attempted by the MAC at a node
- K := At the (K+1)th attempt either the pkt succeeds or is discarded
  - K is the maximum number of retrials
- b_k := The mean back-off at the kth attempt of a pkt, $0 \leq k \leq K$

The Node Response Formula $G(\gamma)$

$$G(\gamma) = \frac{1 + \gamma + \gamma^2 \cdots + \gamma^K}{b_0 + \gamma b_1 + \gamma^2 b_2 + \cdots + \gamma^k b_k + \cdots + \gamma^K b_K}$$

- The formula captures all backoff parameters
- The distribution of the back-off durations does not matter
- No need to keep track of the residual backoff
  - ... as done by Bianchi (and several other researchers)

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Uniqueness Theorem 1

- We have the fixed point equation $\gamma = \Gamma(G(\gamma))$

Lemma

$G(\gamma)$ is nonincreasing in γ if $b_k, k \geq 0$, is a nondecreasing sequence.

- $\Gamma(\beta)$ is increasing in β

Theorem

$\Gamma(G(\gamma)) : [0,1] \rightarrow [0,1]$, has a unique fixed point if $b_k, k \geq 0$, is a nondecreasing sequence.

- In practice, in IEEE 802.11, b_k is a nondecreasing sequence
- The fixed point is then taken as the operating point of the system

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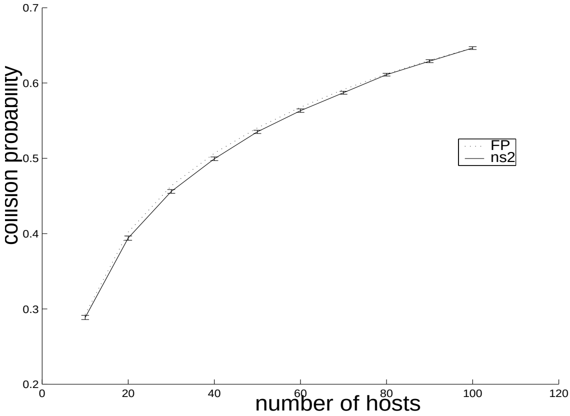
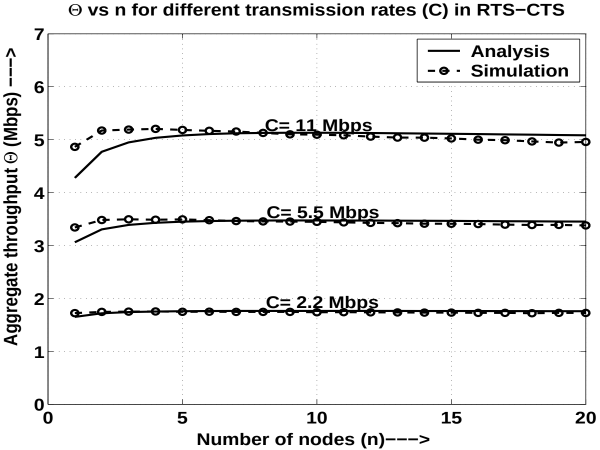
Comparison with Simulation: γ vs. n



- IEEE 802.11 parameters used: $b_0 = 16$ slots, $b_k = 2b_{k-1}, 1 \leq k \leq 7$
- The fixed point analysis provides a very good approximation for a wide range of values of the number of nodes

Comparison with ns2 Simulations



- Note how the saturation throughput is fairly constant with the number of nodes
- Motivates a state dependent service rate model

But Will This Analysis Always Work?

- Is it accidental that it works for the IEEE 802.11 parameters?
- Why does it work?
- Can it be applied to other 802.11-like MACs?
- Next, we provide some partial answers...

Outline of Talk

Homogeneous Nodes: Unbalanced Fixed Points

- A fixed point $(\gamma_1, \gamma_2, \cdots, \gamma_n)$ is called **balanced** if all the coordinates are equal
- A balanced fixed point would be expected to capture the steady state performance
- But can there be unbalanced fixed points?
  - i.e., a vector $(\gamma_1, \gamma_2, \cdots, \gamma_n)$, s.t., for $1 \leq i \leq n$,

$$\gamma_i = 1 - \prod_{j \neq i} (1 - G(\gamma_j))$$

    with all coordinates not equal
- Every permutation of an unbalanced FP is also a FP
- Would suggest multistability or short term unfairness

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Example 1: Homogeneous System, Unbalanced Fixed Points

- There are $n = 10$ nodes

$$G(\gamma) = \frac{1 + \gamma + \gamma^2 + \gamma^3 + \dots}{1 + \gamma + \gamma^2 + \gamma^3 + 64(\gamma^4 + \gamma^5 + \dots)}$$

- i.e., $K = \infty$, $b_0 = b_1 = b_2 = b_3 = 1$, $b_4 = b_5 \cdots = 64$
  - b_0, b_1, b_2, b_3 arise from a point mass at 1
- Intuitively, one node will go on succeeding while all others will be in a long back-off
  - The favoured node will keep changing randomly

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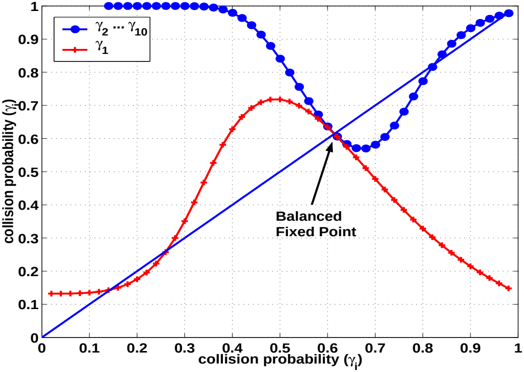
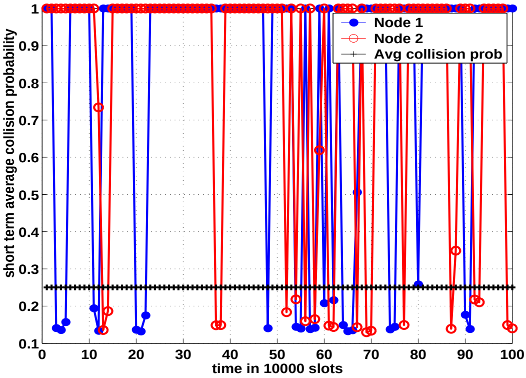
(... contd.) Example 1: Unbalanced FPs

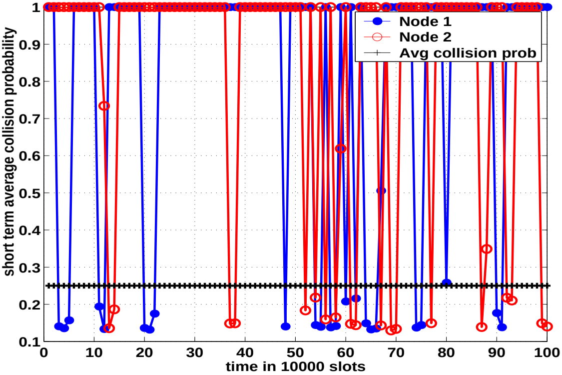


- The balanced fixed point $\gamma_i = 0.62, 1 \leq i \leq 10$, is shown
  - Recall, from earlier theory, that a balanced fixed point must exist
- An unbalanced FP is $\gamma_1 = 0.14, \gamma_i = 0.97, 2 \leq i \leq 10$

(... contd.) Example 1: Multistability



- Short term average collision rate, over 10,000 slot segments
- The node that is being favoured gets a small collision rate
  - while the rest see a collision rate close to 1



- Time average collision rate is 0.25
  - which **does not** match the balanced fixed point: 0.62

Example 2

$$G(\gamma) = \frac{1 + \gamma + \gamma^2 + \cdots + \gamma^7}{1 + 3\gamma + 9\gamma^2 + 27\gamma^3 + \cdots + 2187\gamma^7}$$

- $n = 20$ homogeneous nodes, with the above form of $G(\cdot)$
- This is in the IEEE 802.11 DCF back-off expansion framework
- ... with $K = 7$, $b_0 = 1$, $p = 3$ and $b_k = p^k b_0$ for all $0 \leq k \leq K$
- Again there are multiple nonhomogeneous fixed points
- ... and the balanced fixed point does not capture the steady state performance

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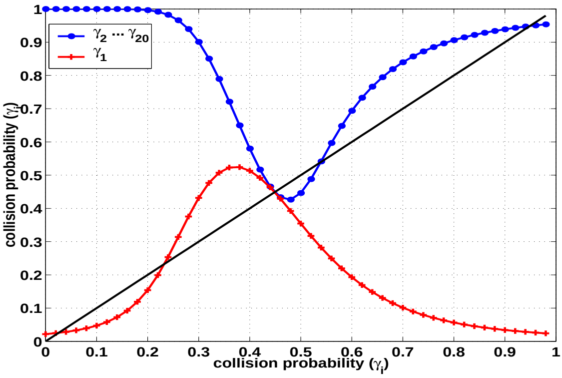
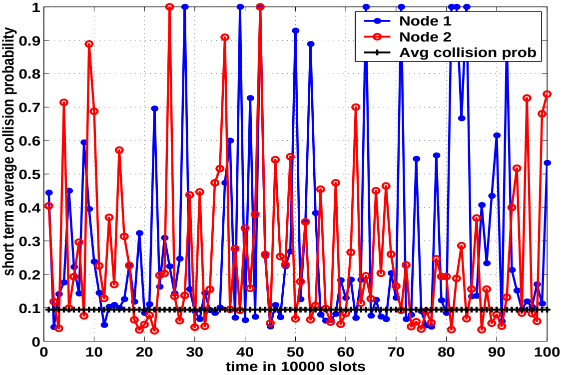


- The balanced fixed point $\gamma_i = 0.45, 1 \leq i \leq 20$, is shown
- Unbalanced FPs are also evident



- Short term average collision rate, over 10,000 slot segments
- The unfairness is less than in Example 1

Uniqueness Theorem 2

$$G(\gamma) = \frac{1 + \gamma + \gamma^2 + \cdots + \gamma^K}{b_0(1 + p\gamma + p^2\gamma^2 + \cdots + p^K\gamma^K)}$$

Theorem

For the function $G(\cdot)$, defined above, if $K \geq 1, p \geq 2$ and $b_0 > 2p + 1$, then the system $\underline{\gamma} = \mathbf{\Gamma}(\mathbf{G}(\underline{\gamma}))$ has a unique fixed point. This fixed point is (of course) balanced.

- Example 2 did not satisfy this sufficient condition
  - ... and we had found that it had unbalanced fixed points

The IEEE 802.11 Standard DCF

- The above result easily extends to the IEEE 802.11 standard where $b_k = p^k b_0$ for $0 \leq k \leq m \leq K$ and $b_k = p^m b_0$ for $m < k \leq K$
- Hence for the IEEE 802.11 standard there is a unique balanced fixed point and there can be no unbalanced fixed points
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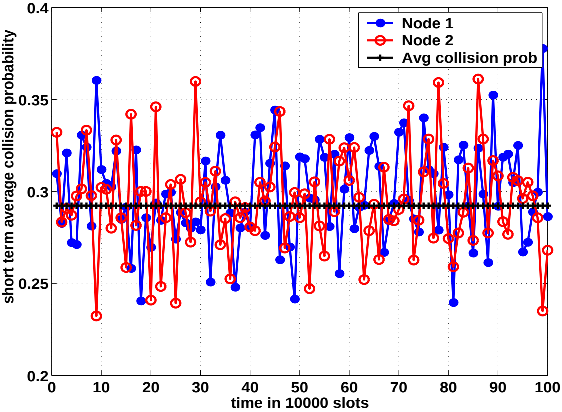
Example 3

$$G(\gamma) = \frac{1 + \gamma + \gamma^2 + \cdots + \gamma^7}{16 + 32\gamma + 64\gamma^2 + \cdots + 2048\gamma^7}$$

- $n = 10$ homogeneous nodes with the above $G(\cdot)$
- Satisfies the condition of the above theorem
  - The parameters are close to those used in IEEE 802.11
- Hence unique fixed point; this fixed point is balanced

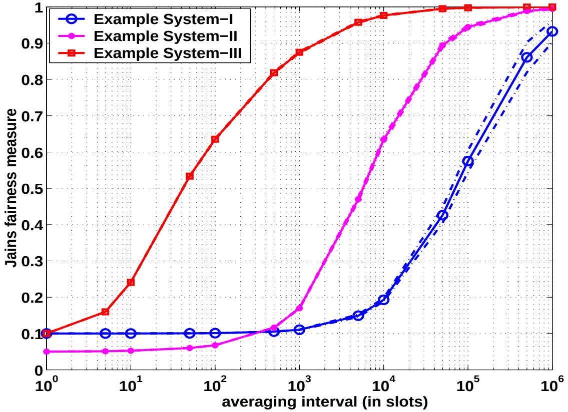
Example 3: Lack of Short Term Unfairness



- Also, the balanced fixed point captures the steady state performance well ($\gamma = 0.29$ from fixed point and from simulation)

Fairness Comparison

- Fairness measure: $\frac{1}{n}\frac{(\sum_{i=1}^{n}\tau_i)^2}{\sum_{i=1}^{n}\tau_i^2}$, where τ_i is the average throughput of node i over the averaging interval



Some Lessons Learnt

- A network with homogeneous nodes can have unbalanced fixed points
- Nonunique fixed points appear to imply multistability
  - ... which manifests itself as unfairness over rather long periods
- When there are nonunique fixed points the balanced fixed point does not capture steady state performance
  - ... which suggests that fixed point analysis will be accurate only when there is a unique, balanced fixed point

Outline of Talk

Uniqueness Theorem 3

- n contending nodes, each with one queue whose $G_i(\cdot)$ function is of the form

$$G_i(\gamma_i) = \frac{1 + \gamma_i + \gamma_i^2 + \cdots + \gamma_i^K}{b_{0_i}(1 + p_i\gamma_i + p_i^2\gamma_i^2 + \cdots + p_i^K\gamma_i^K)}$$

- IEEE 802.11e standard for differentiated services with DCF

Theorem

If $G_i(\cdot)$ satisfies $K_i \geq 1,\ p_i \geq 2$ and $b_{0_i} > 2p_i + 1$, there a exists a unique fixed point for the system of equations $\gamma_i = 1 - \prod_{j \neq i}(1 - G_j(\gamma_j))$ for $1 \leq i \leq n$.

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IEEE 802.11e: The AIFS Mechanism

- In addition to different back-off parameters
  - ... a low priority queue can defer access in favour of higher priority queues
  - ... by waiting longer than DIFS after each channel activity period
- A decoupling approximation leads to a vector fixed point equation

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Comments on the Fixed Point Approach

- Easy to develop the equations and obtain numerical results
  - ... but needs to be used with care, as we have seen
- A formal linkage between the system and the fixed point equations is now emerging
  - Bordenave, McDonald, Proutiere (Allerton 2005): Prove that the back-off stage process decouples in a certain limiting regime
- Such fixed point techniques exist for other problems
  - F.P. Kelly: alternate routing in circuit switched networks
  - A. Stolyar: analysis of input queued packet switches

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